OUNDLE SCHOOL

MAIN ACADEMIC SCHOLARSHIP 2007

MATHEMATICS

PAPER 1

1½ hours.

CALCULATORS ARE NOT ALLOWED FOR THIS PAPER.

INSTRUCTIONS TO CANDIDATES.

You may answer the questions in any order.
You are not expected to have time to do all the questions.
Choose those questions which you think you can answer best.
***Remember to show your working and clearly show the method you are using**.*
Some questions are longer than others.
The number of marks for each question is shown in square brackets.

1. If $u = 5$, $v = -3$, $w = -8$, calculate:

a) $u - v$ b) $w + uv$ c) $5v - 2w$

d) $\dfrac{v+u}{w}$ e) $\dfrac{v^2 - w^2}{u}$ [10]

2. Given that $32.1 \times 98.2 = 3152.22$, calculate the exact values of:

a) 321×0.982 b) $3\,152\,220 \div 9.82$

c) $3.21\times10^4 \times 9.82\times10^3$ d) $3.15222\times10^9 \div (3.21\times10^2)$ [10]

[*Give your answers to (c) and (d) in standard form, i.e. as a number in the form* $a \times 10^n$ *where* $1 \le a < 10$ *and n is an integer.*]

3. Solve: a) $4(x - 3) - 2(1 - 3x) = 7$ b) $x(x - 1) + x(x + 1) = 32$

c) $\dfrac{1}{x} + \dfrac{2}{3} = \dfrac{4}{5}$. [10]

4. Work out as a fraction a) $2 + \dfrac{1}{2 + \frac{1}{2}}$. b) $2 + \dfrac{1}{2 + \dfrac{1}{2 + \frac{1}{2}}}$. [8]

5. Tom loves the moving walkways at the airport. When he walks all the way along one the right way he takes 15 seconds to get to the end. When he goes the whole length the wrong way, he takes 105 seconds. If he walks at 2 m/s, how fast is the walkway going? [8]

6. a) Showing your working, estimate to 1 significant figure $\sqrt{\dfrac{4.86 \times 319}{9.8 - 9.79}}$

b) Calculate $9 \times (8 \times 7 \times (6 - 5) \times 4 - 3 + 2 \times 1)$

c) By considering the last digits of powers of 2 and powers of 5, find the last digit of $2^{34} + 5^{67}$.

d) Find the last digit of 2007^{2007}. [12]

7. a) Twenty years ago the ages of Al, Bob and Carl were in the ratio 1 : 2 : 5. Today the ratio of the ages of Al to Bob is 6 : 7. How old is Carl now?

b) If the point P divides the line AB in the ratio 2 : 5 (i.e. P lies between A and B and $AP : PB$ is 2 : 5), and M and N are one third of the distance along AP and PB respectively, find the ratio $MN : AB$. [11]

8. Six integers have mean 7, median 8, mode 9, and range 10. Find all the possible sets of six numbers for which this is true. [8]

9. a) Jocelyn has a set of five drinks mats, with the numbers 1 to 5 printed on them. She picks out two at random and places them on the table, giving a two-digit number, e.g. 52. Find the probability that:
   i) She produces an even number
   ii) She produces a number divisible by 6.
   b) What would these answers be if she did the same thing with eight mats with the numbers 1 to 8 on them? [9]

10. a) Use algebra to solve the simultaneous equations $3x = 4y$
$3y + x = 13$.
   b) Draw on graph paper two axes each going from 0 to 5 (use 2 cm to 1 unit). Plot the graphs of $y = 4x$, $3x = 4y$ and $3y + x = 13$.
   c) Calculate the exact area of the triangle enclosed by these three lines.
   d) Use part (c) to calculate the exact distance of (1, 4) from the line $3x = 4y$. [18]

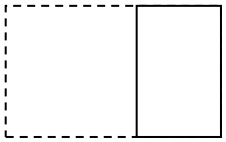
11. The Golden Ratio, ϕ (approximately 1.618), is a number in mathematics with many properties, one of which is that $\phi^2 = \phi + 1$.

   a) Explain why $\phi^3 = 2\phi + 1$

Simplify the following expressions, writing your answer in the form $a\phi + b$, where a and b are integers:

   b) ϕ^5 c) ϕ^{15}

A Golden Rectangle is such that its sides are in the ratio $1:\phi$.



   d) Show that when the largest possible square is cut away from a Golden Rectangle, the remainder is still a Golden Rectangle.

[14]

END OF EXAMINATION